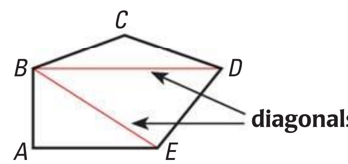
8.1 Find Angle Measures in Polygons

Before	You classified polygons.
Now	You will find angle measures in polygons.
Why?	So you can describe a baseball park, as in Exs. 28–29.

Key Vocabulary
- **diagonal**
- **interior angle,** *p. 218*
- **exterior angle,** *p. 218*

In a polygon, two vertices that are endpoints of the same side are called *consecutive vertices*. A **diagonal** of a polygon is a segment that joins two *nonconsecutive vertices*. Polygon *ABCDE* has two diagonals from vertex *B*, $\overline{BD}$ and $\overline{BE}$.



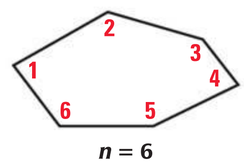
As you can see, the diagonals from one vertex form triangles. In the Activity on page 506, you used these triangles to find the sum of the interior angle measures of a polygon. Your results support the following theorem and corollary.

THEOREMS — *For Your Notebook*

THEOREM 8.1 Polygon Interior Angles Theorem

The sum of the measures of the interior angles of a convex n-gon is $(n - 2) \cdot 180°$.

$m\angle 1 + m\angle 2 + \cdots + m\angle n = (n - 2) \cdot 180°$



Proof: Ex. 33, p. 512 (for pentagons)

COROLLARY TO THEOREM 8.1 Interior Angles of a Quadrilateral

The sum of the measures of the interior angles of a quadrilateral is 360°.

Proof: Ex. 34, p. 512

EXAMPLE 1 Find the sum of angle measures in a polygon

Find the sum of the measures of the interior angles of a convex octagon.

Solution

An octagon has 8 sides. Use the Polygon Interior Angles Theorem.

$(n - 2) \cdot 180° = (8 - 2) \cdot 180°$ **Substitute 8 for *n*.**

$= 6 \cdot 180°$ **Subtract.**

$= 1080°$ **Multiply.**

▶ The sum of the measures of the interior angles of an octagon is 1080°.

EXAMPLE 2 Find the number of sides of a polygon

The sum of the measures of the interior angles of a convex polygon is 900°. Classify the polygon by the number of sides.

Solution

Use the Polygon Interior Angles Theorem to write an equation involving the number of sides n. Then solve the equation to find the number of sides.

$(n - 2) \cdot 180° = 900°$ — Polygon Interior Angles Theorem

$n - 2 = 5$ — Divide each side by 180°.

$n = 7$ — Add 2 to each side.

▶ The polygon has 7 sides. It is a heptagon.

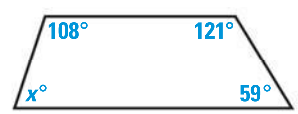
✓ GUIDED PRACTICE for Examples 1 and 2

1. The coin shown is in the shape of a regular 11-gon. Find the sum of the measures of the interior angles.

2. The sum of the measures of the interior angles of a convex polygon is 1440°. Classify the polygon by the number of sides.

EXAMPLE 3 Find an unknown interior angle measure

xy ALGEBRA **Find the value of x in the diagram shown.**



Solution

The polygon is a quadrilateral. Use the Corollary to the Polygon Interior Angles Theorem to write an equation involving x. Then solve the equation.

$x° + 108° + 121° + 59° = 360°$ — Corollary to Theorem 8.1

$x + 288 = 360$ — Combine like terms.

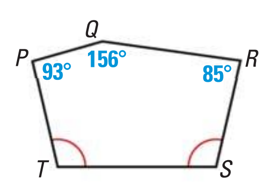
$x = 72$ — Subtract 288 from each side.

▶ The value of x is 72.

✓ GUIDED PRACTICE for Example 3

3. Use the diagram at the right. Find $m\angle S$ and $m\angle T$.

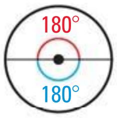
4. The measures of three of the interior angles of a quadrilateral are 89°, 110°, and 46°. Find the measure of the fourth interior angle.

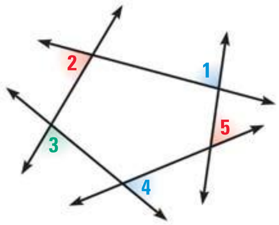


EXTERIOR ANGLES Unlike the sum of the interior angle measures of a convex polygon, the sum of the exterior angle measures does *not* depend on the number of sides of the polygon. The diagrams below suggest that the sum of the measures of the exterior angles, one at each vertex, of a pentagon is 360°. In general, this sum is 360° for any convex polygon.

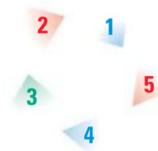
VISUALIZE IT

A circle contains two straight angles. So, there are 180° + 180°, or 360°, in a circle.

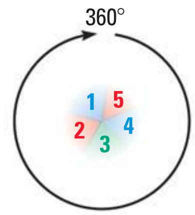




STEP 1 **Shade** one exterior angle at each vertex.



STEP 2 **Cut** out the exterior angles.



STEP 3 **Arrange** the exterior angles to form 360°.

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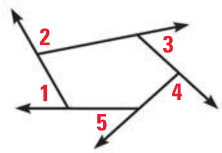
THEOREM — *For Your Notebook*

THEOREM 8.2 Polygon Exterior Angles Theorem

The sum of the measures of the exterior angles of a convex polygon, one angle at each vertex, is 360°.

$m\angle 1 + m\angle 2 + \cdots + m\angle n = 360°$

Proof: Ex. 35, p. 512



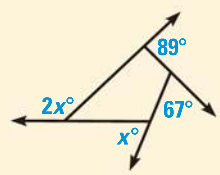
$n = 5$

EXAMPLE 4 Standardized Test Practice

What is the value of x in the diagram shown?

Ⓐ 67 Ⓑ 68

Ⓒ 91 Ⓓ 136



ELIMINATE CHOICES

You can quickly eliminate choice *D*. If x were equal to 136, then the sum of only two of the angle measures ($x°$ and $2x°$) would be greater than 360°.

Solution

Use the Polygon Exterior Angles Theorem to write and solve an equation.

$x° + 2x° + 89° + 67° = 360°$ **Polygon Exterior Angles Theorem**

$3x + 156 = 360$ **Combine like terms.**

$x = 68$ **Solve for *x*.**

▸ The correct answer is B. Ⓐ Ⓑ Ⓒ Ⓓ

✓ GUIDED PRACTICE for Example 4

5. A convex hexagon has exterior angles with measures 34°, 49°, 58°, 67°, and 75°. What is the measure of an exterior angle at the sixth vertex?

EXAMPLE 5 Find angle measures in regular polygons

READ VOCABULARY
Recall that a *dodecagon* is a polygon with 12 sides and 12 vertices.

TRAMPOLINE The trampoline shown is shaped like a regular dodecagon. Find (a) the measure of each interior angle and (b) the measure of each exterior angle.

Solution

a. Use the Polygon Interior Angles Theorem to find the sum of the measures of the interior angles.

$(n - 2) \cdot 180° = (12 - 2) \cdot 180° = 1800°$

Then find the measure of one interior angle. A regular dodecagon has 12 congruent interior angles. Divide 1800° by 12: $1800° \div 12 = 150°$.

▶ The measure of each interior angle in the dodecagon is 150°.

b. By the Polygon Exterior Angles Theorem, the sum of the measures of the exterior angles, one angle at each vertex, is 360°. Divide 360° by 12 to find the measure of one of the 12 congruent exterior angles: $360° \div 12 = 30°$.

▶ The measure of each exterior angle in the dodecagon is 30°.

✓ GUIDED PRACTICE for Example 5

6. An interior angle and an adjacent exterior angle of a polygon form a linear pair. How can you use this fact as another method to find the exterior angle measure in Example 5?

8.1 EXERCISES

HOMEWORK KEY
◯ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 9, 11, and 29
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 18, 23, and 37
◆ = **MULTIPLE REPRESENTATIONS** Ex. 36

SKILL PRACTICE

1. VOCABULARY Sketch a convex hexagon. Draw all of its diagonals.

2. ★ **WRITING** How many exterior angles are there in an n-gon? Are all the exterior angles considered when you use the Polygon Exterior Angles Theorem? *Explain.*

EXAMPLES 1 and 2
on pp. 507–508 for Exs. 3–10

INTERIOR ANGLE SUMS **Find the sum of the measures of the interior angles of the indicated convex polygon.**

3. Nonagon **4.** 14-gon **5.** 16-gon **6.** 20-gon

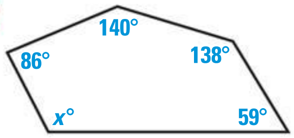
FINDING NUMBER OF SIDES **The sum of the measures of the interior angles of a convex polygon is given. Classify the polygon by the number of sides.**

7. 360° **8.** 720° **9.** 1980° **10.** 2340°

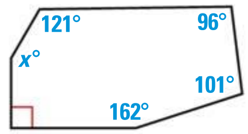
EXAMPLES 3 and 4
on pp. 508–509 for Exs. 11–18

ALGEBRA **Find the value of x.**

11.



12.



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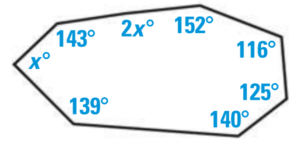

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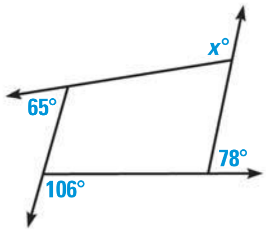

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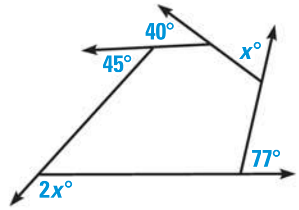
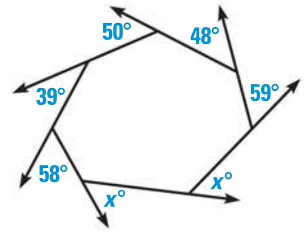


16.



17. ERROR ANALYSIS A student claims that the sum of the measures of the exterior angles of an octagon is greater than the sum of the measures of the exterior angles of a hexagon. The student justifies this claim by saying that an octagon has two more sides than a hexagon. *Describe* and correct the error the student is making.

18. ★ MULTIPLE CHOICE The measures of the interior angles of a quadrilateral are $x°$, $2x°$, $3x°$, and $4x°$. What is the measure of the largest interior angle?

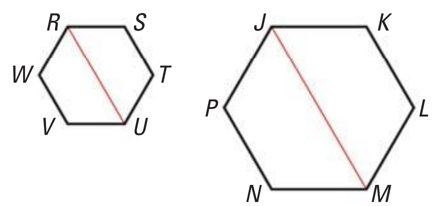
Ⓐ 120° Ⓑ 144° Ⓒ 160° Ⓓ 360°

EXAMPLE 5
on p. 510 for Exs. 19–21

REGULAR POLYGONS **Find the measures of an interior angle and an exterior angle of the indicated regular polygon.**

19. Regular pentagon **20.** Regular 18-gon **21.** Regular 90-gon

22. DIAGONALS OF SIMILAR FIGURES Hexagons *RSTUVW* and *JKLMNP* are similar. $\overline{RU}$ and $\overline{JM}$ are diagonals. Given $ST = 6$, $KL = 10$, and $RU = 12$, find JM.



23. ★ SHORT RESPONSE *Explain* why any two regular pentagons are similar.

REGULAR POLYGONS **Find the value of n for each regular n-gon described.**

24. Each interior angle of the regular n-gon has a measure of 156°.

25. Each exterior angle of the regular n-gon has a measure of 9°.

26. POSSIBLE POLYGONS Determine if it is possible for a regular polygon to have an interior angle with the given angle measure. *Explain* your reasoning.

a. 165° **b.** 171° **c.** 75° **d.** 40°

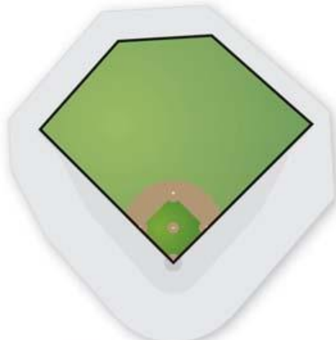
27. CHALLENGE Sides are added to a convex polygon so that the sum of its interior angle measures is increased by 540°. How many sides are added to the polygon? *Explain* your reasoning.

PROBLEM SOLVING

EXAMPLE 1
on p. 507
for Exs. 28–29

BASEBALL The outline of the playing field at a baseball park is a polygon, as shown. Find the sum of the measures of the interior angles of the polygon.

28.



29.

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EXAMPLE 5
on p. 510
for Exs. 30–31

30. JEWELRY BOX The base of a jewelry box is shaped like a regular hexagon. What is the measure of each interior angle of the hexagon?

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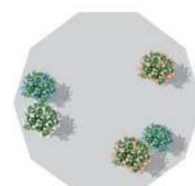
31. GREENHOUSE The floor of the greenhouse shown is a shaped like a regular decagon. Find the measure of an interior angle of the regular decagon. Then find the measure of an exterior angle.



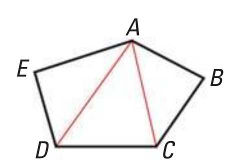
32. MULTI-STEP PROBLEM In pentagon *PQRST*, $\angle P$, $\angle Q$, and $\angle S$ are right angles, and $\angle R \cong \angle T$.

a. Draw a Diagram Sketch pentagon *PQRST*. Mark the right angles and the congruent angles.

b. Calculate Find the sum of the interior angle measures of *PQRST*.

c. Calculate Find $m\angle R$ and $m\angle T$.

33. PROVING THEOREM 8.1 FOR PENTAGONS The Polygon Interior Angles Theorem states that the sum of the measures of the interior angles of an n-gon is $(n - 2) \cdot 180°$. Write a paragraph proof of this theorem for the case when $n = 5$.



34. PROVING A COROLLARY Write a paragraph proof of the Corollary to the Polygon Interior Angles Theorem.

35. PROVING THEOREM 8.2 Use the plan below to write a paragraph proof of the Polygon Exterior Angles Theorem.

Plan for Proof In a convex n-gon, the sum of the measures of an interior angle and an adjacent exterior angle at any vertex is 180°. Multiply by n to get the sum of all such sums at each vertex. Then subtract the sum of the interior angles derived by using the Polygon Interior Angles Theorem.

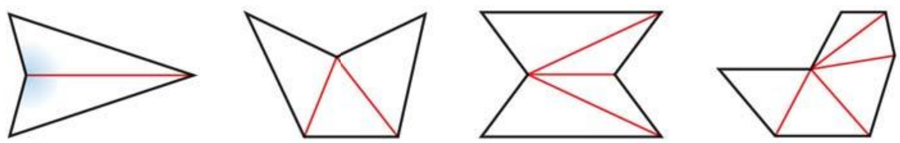
36. **MULTIPLE REPRESENTATIONS** The formula for the measure of each interior angle in a regular polygon can be written in function notation.

a. **Writing a Function** Write a function $h(n)$, where n is the number of sides in a regular polygon and $h(n)$ is the measure of any interior angle in the regular polygon.

b. **Using a Function** Use the function from part (a) to find $h(9)$. Then use the function to find n if $h(n) = 150°$.

c. **Graphing a Function** Graph the function from part (a) for $n = 3, 4, 5, 6, 7$, and 8. Based on your graph, *describe* what happens to the value of $h(n)$ as n increases. *Explain* your reasoning.

37. ★ **EXTENDED RESPONSE** In a concave polygon, at least one interior angle measure is greater than 180°. For example, the measure of the shaded angle in the concave quadrilateral below is 210°.



a. In the diagrams above, the interiors of a concave quadrilateral, pentagon, hexagon, and heptagon are divided into triangles. Make a table like the one in the Activity on page 506. For each of the polygons shown above, record the number of sides, the number of triangles, and the sum of the measures of the interior angles.

b. Write an algebraic expression that you can use to find the sum of the measures of the interior angles of a concave polygon. *Explain*.

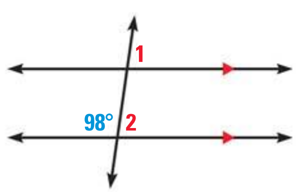
38. **CHALLENGE** Polygon *ABCDEFGH* is a regular octagon. Suppose sides $\overline{AB}$ and $\overline{CD}$ are extended to meet at a point *P*. Find $m\angle BPC$. *Explain* your reasoning. Include a diagram with your answer.

MIXED REVIEW

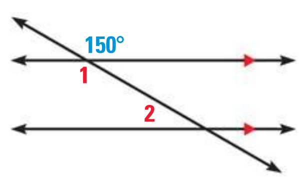
PREVIEW
Prepare for Lesson 8.2 in Exs. 39–41.

Find $m\angle 1$ and $m\angle 2$. *Explain* your reasoning. *(p. 154)*

39.



40.



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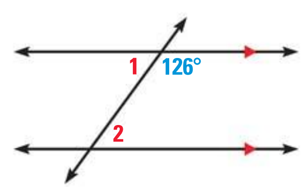

42. Quadrilaterals *JKLM* and *PQRS* are similar. If $JK = 3.6$ centimeters and $PQ = 1.2$ centimeters, find the scale factor of *JKLM* to *PQRS*. *(p. 372)*

43. Quadrilaterals *ABCD* and *EFGH* are similar. The scale factor of *ABCD* to *EFGH* is 8 : 5, and the perimeter of *ABCD* is 90 feet. Find the perimeter of *EFGH*. *(p. 372)*

Let $\angle A$ be an acute angle in a right triangle. Approximate the measure of $\angle A$ to the nearest tenth of a degree. *(p. 483)*

44. $\sin A = 0.77$ **45.** $\sin A = 0.35$ **46.** $\cos A = 0.81$ **47.** $\cos A = 0.23$